UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Subsidiary Level

MATHEMATICS **9709/23**

Paper 2 Pure Mathematics 2 **(P2)** **May/June 2010**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

7552068932

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **3** printed pages and **1** blank page.

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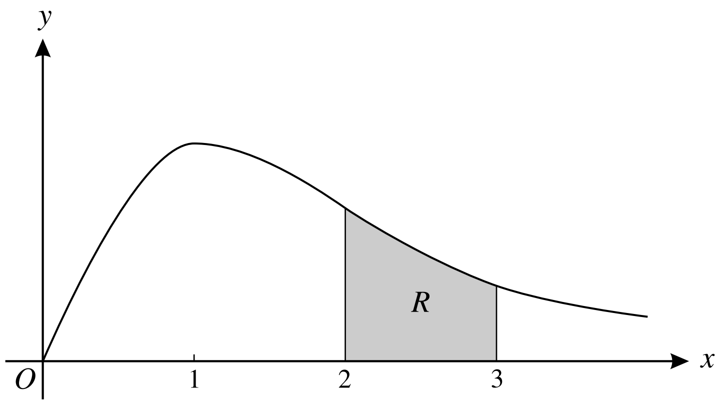
1 Given that $13^x = (2.8)^y$, use logarithms to show that $y = kx$ and find the value of k correct to 3 significant figures. [3]

2



The diagram shows part of the curve $y = xe^{-x}$. The shaded region R is bounded by the curve and by the lines $x = 2$, $x = 3$ and $y = 0$.

(i) Use the trapezium rule with two intervals to estimate the area of R, giving your answer correct to 2 decimal places. [3]

(ii) State, with a reason, whether the trapezium rule gives an under-estimate or an over-estimate of the true value of the area of R. [1]

3 Solve the inequality $|2x - 1| < |x + 4|$. [4]

4 **(a)** Show that $\int_0^{\frac{1}{4}\pi} \cos 2x \, dx = \frac{1}{2}$. [2]

(b) By using an appropriate trigonometrical identity, find the exact value of

$$\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} 3\tan^2 x \, dx.$$ [4]

5 The equation of a curve is $y = x^3 e^{-x}$.

(i) Show that the curve has a stationary point where $x = 3$. [3]

(ii) Find the equation of the tangent to the curve at the point where $x = 1$. [4]

6 **(i)** By sketching a suitable pair of graphs, show that the equation

$$\ln x = 2 - x^2$$

has only one root. [2]

(ii) Verify by calculation that this root lies between $x = 1.3$ and $x = 1.4$. [2]

(iii) Show that, if a sequence of values given by the iterative formula

$$x_{n+1} = \sqrt{(2 - \ln x_n)}$$

converges, then it converges to the root of the equation in part **(i)**. [1]

(iv) Use the iterative formula $x_{n+1} = \sqrt{(2 - \ln x_n)}$ to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

7 The polynomial $2x^3 + ax^2 + bx + 6$, where a and b are constants, is denoted by $\mathrm{p}(x)$. It is given that when $\mathrm{p}(x)$ is divided by $(x - 3)$ the remainder is 30, and that when $\mathrm{p}(x)$ is divided by $(x + 1)$ the remainder is 18.

(i) Find the values of a and b. [5]

(ii) When a and b have these values, verify that $(x - 2)$ is a factor of $\mathrm{p}(x)$ and hence factorise $\mathrm{p}(x)$ completely. [4]

8 **(i)** Prove the identity

$$\sin(x - 30^\circ) + \cos(x - 60^\circ) \equiv (\sqrt{3}) \sin x. \quad [3]$$

(ii) Hence solve the equation

$$\sin(x - 30^\circ) + \cos(x - 60^\circ) = \tfrac{1}{2} \sec x,$$

for $0^\circ < x < 360^\circ$. [6]

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